Michael Dang – 16257750

MATH345L

Final Project



* The Euler method is a first-order numerical procedure for solving ordinary differential equation (ODEs) with a given initial value. It is a straight-forward method that estimates the next point based on the rate of change at the current point and it is easy to code. It is a single step method. However, Euler's method is unconditionally unstable for un-damped oscillating systems. For complex problems and/or boundary condition it may fail. It can be used for basic numerical analysis.
* The forward Euler method is the simplest RK (Runge-Kutta) method (1 stage, first order). Higher order accurate RK methods are multi-stage because they involve slope calculations at multiple steps at or between the current and next discrete time values. The next value of the dependent variable is calculated by taking a weighted average of these multiple stages based on a Taylor series approximation of the solution. The most popular RK method is RK4 since it offers a good balance between order of accuracy and cost of computation. RK4 is the highest order explicit RK method that requires the same number of steps as the order of accuracy. Beyond fourth order the RK methods become relatively more expensive to compute.

1. c.

Table 1, Euler’s method with h = 0.05

|  |  |  |  |
| --- | --- | --- | --- |
| t | Approximated Sol | Exact Sol | Absolute error |
| 0.2 | 1.4521 | 1.4371 | 0.0150 |
| 0.4 | 1.7835 | 1.7565 | 0.0270 |
| 0.6 | 2.0060 | 1.9694 | 0.0366 |
| 0.8 | 2.1300 | 2.0858 | 0.0442 |
| 1.0 | 2.1651 | 2.1151 | 0.0500 |

Table 2, Euler’s method with h = 0.1

|  |  |  |  |
| --- | --- | --- | --- |
| t | Approximated Sol | Exact Sol | Absolute error |
| 0.2 | 1.4675 | 1.4371 | 0.0304 |
| 0.4 | 1.8114 | 1.7565 | 0.0549 |
| 0.6 | 2.0438 | 1.9694 | 0.0744 |
| 0.8 | 2.1755 | 2.0858 | 0.0897 |
| 1.0 | 2.2164 | 2.1151 | 0.1013 |

Table 3, Euler’s method with h = 0.025

|  |  |  |  |
| --- | --- | --- | --- |
| t | Approximated Sol | Exact Sol | Absolute error |
| 0.2 | 1.4445 | 1.4371 | 0.0074 |
| 0.4 | 1.7699 | 1.7565 | 0.0134 |
| 0.6 | 1.9876 | 1.9694 | 0.0182 |
| 0.8 | 2.1078 | 2.0858 | 0.0220 |
| 1.0 | 2.1399 | 2.1151 | 0.0248 |

1. c

Table 1, Runge-Kutta’s method with h = 0.05

|  |  |  |  |
| --- | --- | --- | --- |
| t | Approximated Sol | Exact Sol | Absolute error |
| 0.2 | 1.4371 | 1.4371 | 0.0000 |
| 0.4 | 1.7203 | 1.7565 | 0.0363 |
| 0.6 | 1.9175 | 1.9694 | 0.0519 |
| 0.8 | 2.0199 | 2.0858 | 0.0659 |
| 1.0 | 2.0364 | 2.1151 | 0.0787 |

Table 2, Runge-Kutta’s method with h = 0.1

|  |  |  |  |
| --- | --- | --- | --- |
| t | Approximated Sol | Exact Sol | Absolute error |
| 0.2 | 1.4371 | 1.4371 | 0.0000 |
| 0.4 | 1.6839 | 1.7565 | 0.0725 |
| 0.6 | 1.8657 | 1.9694 | 0.1037 |
| 0.8 | 1.9539 | 2.0858 | 0.1318 |
| 1.0 | 1.9577 | 2.1151 | 0.1574 |

Table 3, Runge-Kutta’s method with h = 0.025

|  |  |  |  |
| --- | --- | --- | --- |
| T | Approximated Sol | Exact Sol | Absolute error |
| 0.2 | 1.4371 | 1.4371 | 0.0000 |
| 0.4 | 1.7384 | 1.7565 | 0.0181 |
| 0.6 | 1.9434 | 1.9694 | 0.0260 |
| 0.8 | 2.0529 | 2.0858 | 0.0329 |
| 1.0 | 2.0758 | 2.1151 | 0.0393 |